Philadelphia University

Lecture Notes for 650364

# Probability & Random Variables

## Chapter 1:

## Lecture 1: Introduction and Set Theory

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# Probability

1) Introduction

2) Set Definitions

3) Set Operations

4) Probability Introduced Through Sets and Relative Frequency

5) Joint and Conditional Probability

6) Total Probability and Bayes' Theorem

7) Independent Events

8) Combined Experiments

9) Bernoulli Trials

# 1)Introduction

- ✓ The **primary goals** are:
  - To introduce the principles of **random signals**.
  - To provide **tools** whereby one can deal with systems involving such signals.
- ✓ A **random signal** is a time waveform that can be characterized in some **probabilistic** manner.
  - **Examples**: broadcast radio receiver, different types of noises, TV system and its noises, sonar system randomly generated sea sounds and bits in a computer bit stream.
- ✓ **Probability** can be defined as the **mathematics of chance**.
- ✓ **Probability** is a field of mathematics, which investigates the behavior of mathematically defined random experiments
- ✓ **Combining events**: We can form new events from events by using logical rules: Let A, B are some events
  - "A and B occur"
  - "A or B occur"
  - "A does not occur"

- "B occurs, but A does not"

✓ In order to determine the probabilities, we must express the events using **set theory.**

## 2) Set Definitions

✓ A **set** is a **collection** of objects called **elements** of the set.

- **Examples**: set of voltages, set of numbers.
- A ***set of sets*** sometimes called **class** of sets.

✓ A set is usually denoted by a **capital letter** while an element is represented by a **lower-case letter.**

- If $a$ is an element of set $A$, then we write

$$a \in A$$

- If $a$ is not an element of set $A$, then we write

$$a \notin A$$

✓ A set is specified by the content of two braces: **{.}**

✓ Two methods exist for specifying content:

- The **tabular** method.
- The **rule** method.

- **Examples**:
  - The set of all integers between 5 and 10 would be:
    - In tabular method such as:
      - **A = {6,7,8,9}.**
      - **S = {book, cell phone, mp3, paper, laptop}**
    - In rule method such as: **{integers between 5 and 10}**
- A set with a large or infinite number of elements are best described by a statement or rule method. for example:
    - **{Integers from 5 to 10000 inclusive}**
    - **S = {x | x is a city with a population over 1million}**

✓ **Countable and Uncountable Sets**:

- A set is called **countable** if its elements can be put in one-to-one correspondence with **natural numbers.**
    - **Examples**:
      - **A = {1, 3, 5, 7}**
      - **S={H,T}**
      - **B = {1, 2, 3, ….}**
    - A set is called **uncountable,** if its elements not countable

- **Examples:**
  - $C = \{0.5 < c \leq 8.5\}$
  - $D = \{-0.5 < d \leq 12.0\}$
  - $S = \{t \mid t > 0\}$

- **Empty** set or **null** set is a set, which contains no elements, at all, denoted by the symbol **Ø** and written as **{ }.**

✓ A **finite set**: is either empty or has elements that can be counted.

- **Examples:**
  - $A = \{1, 3, 5, 7\}$
  - $D = \{0.0\}$ ← **not the null set, it has one element**
  - $E = \{2, 4, 6, 8, 10, 12, 14\}$

✓ If a set is not finite it is called **infinite**.

- **Examples:**
  - $B = \{1, 2, 3, \ldots\}$
  - $C = \{0.5 < c \leq 8.5\}$
  - $D = \{-0.5 < d \leq 12.0\}$

- ✓ The set A is called a **subset** of B if every element in A is also an element in B (**A contained in B**), we write

$$A \subseteq B$$

- ✓ If at least one element exists in B which is not in A, then A is a **proper** subset of B, denoted by

$$A \subset B$$

- ✓ The null set is clearly a subset of all other sets.
- ✓ Two sets A and B is called **disjoint** or **mutually exclusive** if they have no common elements :

$$A \cap B = AB = \emptyset$$

- **Examples:**

$A = \{1, 3, 5, 7\}$ $B = \{1, 2, 3, \ldots\}$

$C = \{0.5 < c \leq 8.5\}$ $D = \{0.0\}$

$E = \{2, 4, 6, 8, 10, 12, 14\}$ $F = \{-5.0 < f \leq 12.5\}$

- **A: tabular-specified countable and finite.**
- **B: is also tabular-specified and countable but infinite.**
- **C: rule- specified, uncountable and infinite.**
- **D and E are mutually exclusive.**
- **F is uncountable and infinite**

- **Set A is contained in set B, C and F.**
- $C \subset F\ ,\ D \subset F\ ,\ E \subset B$
- **Sets A, D and E are mutually exclusive.**

✓ The largest set of objects under discussion in a given situation is called the **universal set**, denoted **S**.

- **Examples**: In the problem of rolling a die, we are interested in the numbers that show on the upper face. The universal set is

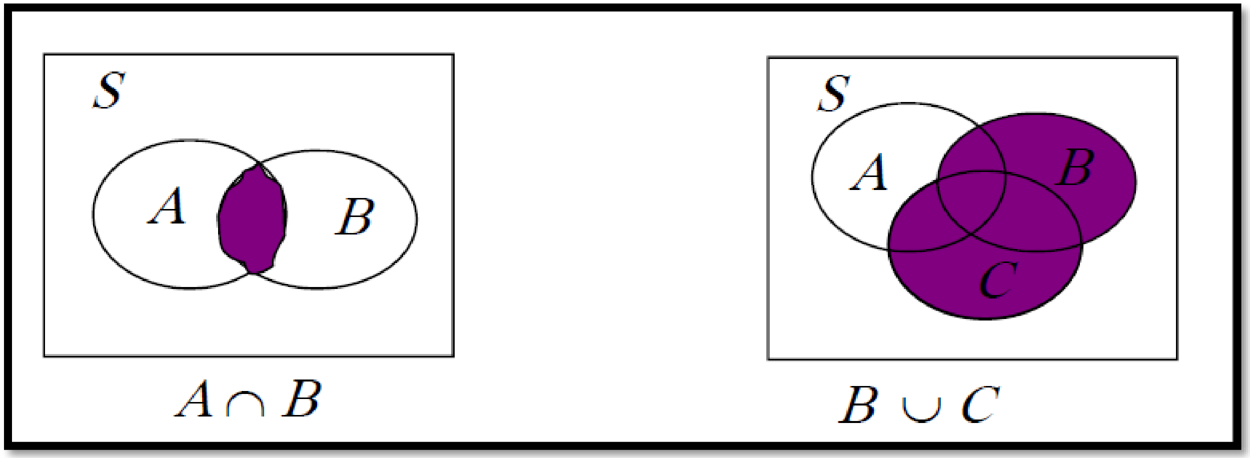
  **S = {1,2,3,4,5,6}**

✓ For any universal set with N elements, there are $2^N$ possible subsets of **S.**

## 3)Set Operations

✓ **Geometrical representation** of the sets using **Venn diagram**. The relationship between subsets and the universal set can be illustrated graphically using **Venn diagram.**

- Sets are represented by closed-plane figures.




✓ **Equality**: Two sets A and B are equal if and only if they have the same elements. We write $A = B$

$$A \subseteq B \text{ and } B \subseteq A$$

✓ **Difference**: The difference of two sets A and B, is the set containing all elements of A that are not present in B. We write

$$A - B$$

- **Example:**

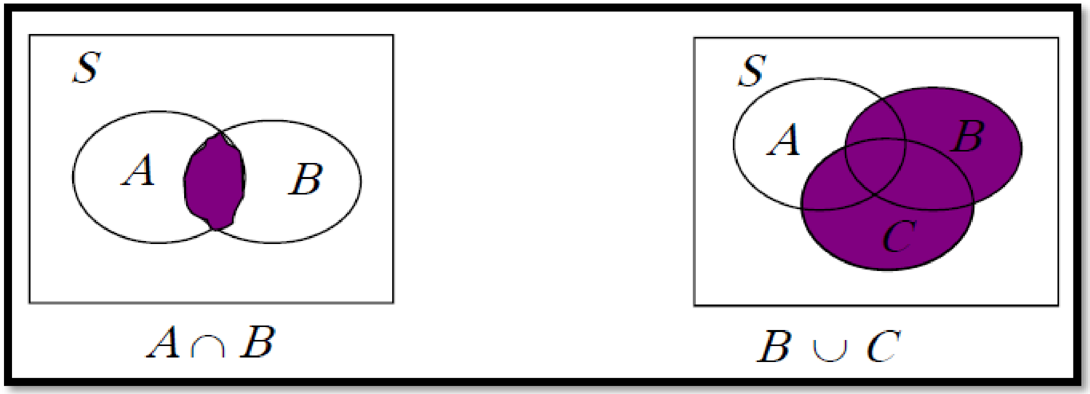
If $A = \{0.6 < a \leq 1.6\}$ and $B = \{1.0 \leq b \leq 2.5\}$

Then $A - B = \{0.6 < c < 1.0\}$

$$B - A = \{1.6 < c \leq 2.5\}$$

- Note that $A - B \neq B - A$

- ✓ The **union** of two sets A and B, written as $C = A \cup B$, is the set containing all elements of both A and B or both, Union sometimes called the **sum** of two sets.
- ✓ The **intersection** of two sets A and B, written as $D = A \cap B$, is the set of all elements common to both A and B. Intersection sometimes called the **product** of two sets.
  - For mutually exclusive sets A and B, $A \cap B = \emptyset$




- ✓ In general case, the union and intersection of N sets An, n=1,2,3,….,N , become:

$$A_1 \cup A_2 \cup \cdots \cup A_N = \bigcup_{n=1}^{N} A_n$$

$$A_1 \cap A_2 \cap \cdots \cap A_N = \bigcap_{n=1}^{N} A_n$$

- ✓ The **complement** of set A, denoted by $\bar{A}$, is the set of all elements not in A.
  - Note that:

$$\overline{S} = \Phi, \quad \overline{\Phi} = S, \quad A \cup \overline{A} = S \quad \text{and} \quad A \cap \overline{A} = \Phi$$

- ✓ **Example**:

Given the four sets:

$$S = \{1 \le integers \le 12\}$$
$$A = \{1, 3, 5, 12\}$$
$$B = \{2, 6, 7, 8, 9, 10, 11\}$$
$$C = \{1, 3, 4, 6, 7, 8\}$$

Then

$$A \; U \; B = \{1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$A \cup C = \{1, 3, 4, 5, 6, 7, 8, 12\}$$
$$B \cup C = \{1, 2, 3, 4, 6, 7, 8, 9, 10, 11\}$$
$$A \cap B = \emptyset , A \cap C = \{1, 3\} , B \cap C = \{6, 7, 8\}$$
$$\overline{A} = \{2, 4, 6, 7, 8, 9, 10, 11\}$$
$$\overline{B} = \{1, 3, 4, 5, 12\}$$
$$\overline{C} = \{ 2, 5, 9, 10, 11, 12\}$$

✓ **Algebra of Sets**:

- **Commutative Law:**

$$A \cap B = B \cap A$$
$$A \cup B = B \cup A$$

- **Distributive Law:**

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

- **Associative Law:**

$$(A \cup B) \cup C = A \cup (B \cup C) = A \cup B \cup C$$
$$(A \cap B) \cap C = A \cap (B \cap C) = A \cap B \cap C$$

- **De Morgan's Law:**

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$
$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

Replace unions by intersections, intersections by unions, by use of a venn

- **Duality Principle:**

In any an identity we replace unions by intersections, intersections by unions, **S** by **Ø**, and **Ø** by **S**, then the identity is preserved.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$